

A Higher Calling: Career Concerns and the Number of Political Parties*

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Abstract

It is generally accepted that first-past-the-post elections induce a tendency towards only two parties competing in a given election. What is less clear is why we should see the same two parties competing in separate districts or at different levels of government, as it is the case in the US. In fact, it seems puzzling that no third parties are able to enter successfully in the US, given the almost complete lack of competition in some states. This paper proposes the career concerns of politicians as an explanation and demonstrates this in a novel model of party formation: state politicians would like to advance their career to the federal level, but only have the opportunity of doing so as a member of a federally successful party. If politicians value such career opportunities sufficiently strongly, entry of additional parties at the state level does not occur. There then exists an equilibrium with two parties, one centre-left and one centre-right, where each party dominates some states. When career concerns are weak, on the other hand, the number of parties in equilibrium will be at least equal to three with a tendency towards parties with a narrower ideological profile.

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JEL Classification: D72.

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1 Introduction

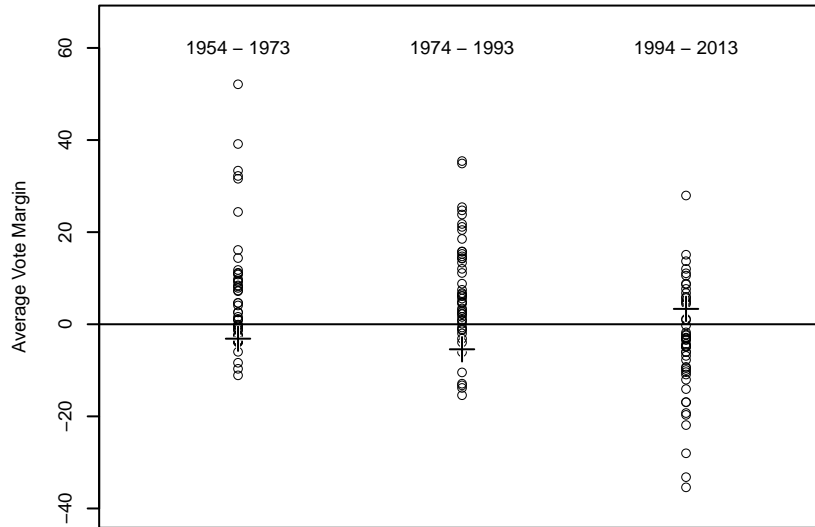
Duverger's famous law states that first-past-the-post elections (FPTP) in combination with single member districts should lead to competition among two parties, and is frequently cited to explain the persistent dominance of the Democratic and the Republican Party in US politics. The logic underlying this claim is that losing parties will either be abandoned by voters or will decide by themselves to drop out of the race until only two parties remain. As has been recognized (Cox 1994), this line of reasoning applies to a single election, but not to elections held across separate districts or at multiple levels of government. Applying Duverger's law to the US, for example, we should expect to see two parties competing for the governorship of California and two parties competing for the Presidency, but there is no reason why the same two parties should be competing in both of these elections. In fact, it seems surprising that there are only two effective parties observable once one takes a look at some broad patterns in election outcomes. Figure 1 displays average differences in the vote share of Democratic and Republican candidates in presidential and gubernatorial elections across three 20-year periods.¹ The figure clearly shows that the outcomes of presidential elections are generally fairly close and do not consistently favour one party. Under these conditions the chances of any third party successfully contesting the presidential election indeed seem slim. In contrast, however, there is no lack of states where the candidates of one party consistently win elections with margins of victory of above 20 percent. In some extreme cases, margins of victory in gubernatorial elections approach 40 percent *on average*. Why are no third parties able to exploit this lack of competition? This question seems particularly relevant since other countries relying on FPTP elections, such as Canada, do feature different parties active at the regional and at the national level.²

In this paper I propose a new determinant of the number of parties that can explain the persistent duopoly in US politics, namely the career concerns of politicians: Federally successful parties can prevent members at the state level from defecting by offering career prospects at the federal level. I illustrate the strength of this logic in a novel model of party formation that puts no ad-

¹Considering these elections has the advantage that they are not influenced by gerrymandering.

²Of course this comparison is not perfect, as there are important differences in the political systems of Canada and the US. Most notably, Canada has a parliamentary system while the US has a presidential system.

Figure 1: Average Differences in Vote Shares of Democratic and Republican Candidates



Notes: Each circle represents gubernatorial elections in a given state, while crosses stand for presidential elections. In the latter case, the numbers are based on popular vote shares.

Sources: Presidential elections - www.ropercenter.uconn.edu/elections/common/pop_vote.html; Gubernatorial elections up to 1990 - ICPSR (1995); Gubernatorial elections after 1990 - library.cqpress.com/elections/

hoc limits on the number of parties existing in equilibrium. The main result of the paper shows that equilibria with two parties exist only if politicians' career concerns are sufficiently strong. If politicians are mostly motivated by opportunities at the state level, in contrast, any equilibrium must feature three or more parties.

In order to explain the logic underlying my results more clearly, let me first provide some details about the model. Given the question at hand, the model naturally features elections for state governments as well as a federal election. Candidates for all of these elections are nominated by political parties. Politicians standing at the beginning of their career join these parties in order to signal their policy preferences to voters. Parties enable politicians to do

so by allowing only certain types of politicians to join. Parties thus serve as “informative labels” (Snyder & Ting 2002) that provide information about their members to voters. A politician who has won a state election then has a chance to become their party’s candidate for the federal election.

A crucial feature of the model, which is also consistent with the data presented in Figure 1, is that there is a minimum amount of heterogeneity in voter preferences across states. This forces parties to adopt a broad ideological profile if they want to cater to voters’ diverse tastes and prevent entry of additional parties. But if parties allow a wide range of politicians to join, this creates intense internal competition for the party nomination in the run-up to state elections. Politicians can be willing to put up with this competition if they see the state election as a mere stepping stone towards more attractive positions at the federal level. However, if politicians do not value such opportunities much, they will be willing to join smaller, more ideologically target parties that feature less internal competition. The key to the main result is then to show that any constellation of two parties is vulnerable to entry of such smaller parties, in which case two parties can only be an equilibrium if career concerns are sufficiently strong.

Beyond this general result I also provide existence conditions for a particular two-party equilibrium with a centre-left and a centre-right party that looks very similar to what we observe in the US. In the context of the model, this equilibrium seems natural. A wider overlap among the sets of politicians allowed to join each party would create stronger internal competition and might lead to defections. A gap between parties, on the other hand, could create an opportunity for entry of a centrist party. In addition, this equilibrium recreates the pattern in Figure 1: States with extreme median voters vote overwhelmingly in favour of one party, resulting in wide vote margins. States with centrist median voters, in contrast, are more competitive. The federal election, finally, is competitive as both parties generate symmetric candidate pools centred around the federal median voter. While a simpler model with a fixed number of parties might also be able to replicate this pattern in election results, such a model would clearly not be able to answer the deeper question about why we see the same parties competing across different geographic levels. The main contribution of the paper is therefore to establish the career concerns of politicians as an explanation for the lack of entry at the state level and as a driver of party formation more generally.

A concurrent and independent paper by Aldrich & Lee (2016) also highlights

the importance of political ambitions in explaining why only two parties exist in the US. To make this point, these authors discuss a utility function for politicians and explain how the utility of joining a party that offers the highest probability of winning a state election can be lower than joining a national party as long as the national party offers a sufficiently high probability of winning elections at the federal level. This utility function is not embedded in an equilibrium model and there is no explanation why the chances of winning the state election should be lower as a member of the national party in the first place. In the current paper national parties are less attractive due to intense internal competition for nominations, which arises endogenously. In addition, I show that heterogeneity in voter preferences is another crucial ingredient: If the median voter had the same position in all elections, an equilibrium with a single party would exist in my model even if politicians do not care about winning federal elections at all.

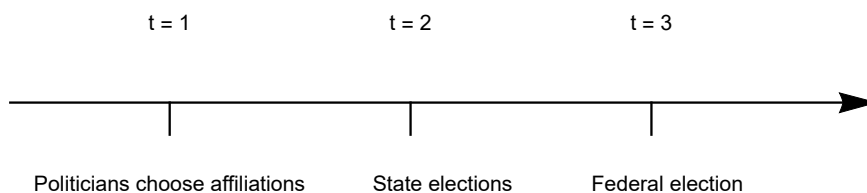
This paper is related to the literature on political competition with entry (Palfrey 1984, Osborne 1993, 2000, Callander 2005), which analyses the effect that the threat of entry has on the equilibrium behaviour of two parties. Perhaps closest to the current paper is Callander (2005), who studies competition between two parties in multiple single-member districts with threat of entry at the district level. Parties, which are not explicitly modelled, are free to choose any platform. Callander finds that the threat of entry leads to the divergence of party platforms, similar to this paper. The mechanism through which entry is deterred is different though. In addition, the equilibrium presented by Callander requires very specific assumptions on the distribution of voters across districts, while the restrictions imposed on voter distributions in this paper are mild. Eyster & Kittsteiner (2007), on the other hand, present a model that features multiple districts, but take the number of parties as fixed. Neither of these papers mentions career concerns.

Political parties clearly form a central element of the political system of any democratic country, yet they have received surprisingly little attention, at least in terms of formal modelling. Few papers have attempted to fully endogenise the number parties existing in equilibrium as I do here (Jackson & Moselle 2002, Levy 2004, Morelli 2004, Osborne & Tourky 2008, Eguia 2011). These papers typically focus on policy-motivated politicians, while the internal party politics of my model are largely driven by office motivations.³

As mentioned above, the concept of political parties that I employ is taken

³The politicians in Morelli (2004) are both office- and policy-motivated, but this only influences their choice of whether to run for office as they do not choose their party affiliation.

Figure 2: Timing



from Snyder & Ting (2002). These authors, as well as other contributions building on their approach (Ashworth & Bueno de Mesquita 2008, Bernhardt et al. 2009), consider the behaviour of a given number of parties. I show how the concept of parties as “informative labels” can yield an equilibrium with two parties that closely resembles reality even after endogenising the number of parties.

The rest of the paper is organized as follows: Section 2 explains the details of the model, while Section 3 gives the theoretical results. Robustness of the results to relaxing some of the assumptions made in the basic version of the model is discussed in Section 4. Section 5 concludes.

2 The Model

A federal state consisting of $S \geq 4$ states selects federal and state governments through FPTP elections. Candidates for these elections are nominated by political parties. Initially a large number of potential parties exists, but only those that manage to attract members can compete in elections. The timing, summarized in Figure 2, is as follows: In the beginning of the game politicians decide which party to join. Once affiliation decisions have been made, parties nominate candidates in each state and state elections are held. Each winner of a state election then has a chance to become their party’s candidate for the federal election. After the federal election the game ends.

2.1 Voters, Politicians, and Parties

Each state s has a set of citizens that is large, finite, and odd. Each citizen votes in two elections: the election for the government of state s and the election for the federal government. Let p_s and p_f denote the policies that are implemented in state s and at the federal level, respectively. The objective of voters in an election in region $r \in \{1, \dots, S, f\}$ is to maximize

$$\mathbb{E}[u(|p_r - i|)] ,$$

where $u : \mathbb{R}_+ \rightarrow \mathbb{R}$ is a concave function while $i \in \mathbb{R}$ is the ideal policy of the voter.⁴

Each state also has a finite set of politicians. Every politician is endowed with a fixed platform and once elected to any office a politician is committed to implementing this platform. The platform of a politician is supposed to represent the ideal policy of a politician. Preferences over policies appear to be the main driver of the choices that politicians make in office (Levitt 1996, Chattopadhyay & Duflo 2004, Lee et al. 2004, Bhalotra & Clots-Figueras 2014). Of course it would be preferable that this behaviour emerges as part of an equilibrium, rather than being imposed from the outset. I will allow politicians to be more flexible in their policy choices in Section 4.2.

The set of possible platforms is given by $\mathcal{T} \subset [-2, 2]$ with $\{-1, 0, 1\} \subseteq \mathcal{T}$. Platforms are evenly distributed such that there is the same distance between any two adjacent platforms and their total number is odd. Each state has $|\mathcal{T}|$ politicians, none of which share the same platform. Put differently, there is one politician located at each of the possible platforms.

Politicians who do not join a party receive a payoff of zero. The payoff from joining a party, on the other hand, depends on the electoral success of the party. If a politician joins a party that does not win a single election, she incurs a cost $y_z < 0$. If a party wins at least one election, each member receives a payoff $y_w > 0$. These payoffs can be thought of as the psychological costs or benefits of being on a losing/winning team and may be arbitrarily small. Of much greater importance for the analysis are the payoffs associated with personally winning elected office. The winning candidate in an election at the state level receives a payoff of $y_s > 0$, while the utility of the winning candidate

⁴As will become clear later, the outcomes of state elections may affect events at the federal level, but it is assumed that voters do not take this interdependence into account when voting at the state level.

at the federal election further increases by $y_f > 0$. These payoffs subsume the material and immaterial benefits of holding office. For example, purely local concerns that might motivate a politician could form part of y_s . The strength of politicians' career concerns is captured by the ratio of the payoffs y_f and y_s . The larger this ratio, the more politicians are driven by the pursuit of success at the federal level.

In order to clearly define the utility of a politician who joins a party that wins at least one election, let π_s be the probability that a politician is nominated for and wins the state election in her state. Conditional on doing so, let π_f give the probability that a politician is nominated for and wins the federal election. Both these probabilities will later be determined in equilibrium. The expected utility of a politician who has joined a party that wins at least one election is then given by

$$y_w + \pi_s(y_s + \pi_f y_f) .$$

As was mentioned before, politicians have to join parties in order to win elections. A political party is basically a subset of the policy space and only politicians whose platforms fall within this subset can join. One way to think of this is that parties can screen their members and exclude those whose platforms do not agree with the party line. This concept of parties is based on Snyder & Ting (2002). Formally, a party is an interval $[a, b]$ with $\{a, b\} \subset \mathcal{T}$. If $a = b$ I simply write $[a]$. The set of all possible shapes parties can have is given by $\mathcal{I} = \{[a, b] | a, b \in \mathcal{T}\}$. Individual parties will be denoted by capital letters and for any such party P the interval representing the party is given by I_P . \mathcal{P} is the set of potential parties. That is, each element P of \mathcal{P} is a party with shape $I_P \in \mathcal{I}$ and for each possible shape $J \in \mathcal{I}$ there exists at least one party $P \in \mathcal{P}$ such that $I_P = J$. Denote by $\mathcal{P}(p)$ the set of parties that allow politicians with platform p to join. Formally,

$$\mathcal{P}(p) \equiv \{P \in \mathcal{P} | p \in I_P\} .$$

The strategy space of a politician with platform p is given by $\mathcal{P}(p) \cup \{\emptyset\}$, where \emptyset represents the choice not to join a party. Thus, politicians form parties by coordinating on joining one of the potential parties available in \mathcal{P} . Parties that have attracted at least one member will be referred to as active parties. Due to the negative payoff y_z of being a member of a party that does not win any elections, any active party must also win at least one election in equilibrium.

Immediately prior to each state election every party that is active in the state nominates a candidate. All members of a party within a state are eligible and each one of them is nominated with equal probability. Denoting by $\mathcal{M}_{P,s}$ the set of politicians who has joined party P in state s , each member of $\mathcal{M}_{P,s}$ is thus nominated with probability $1/|\mathcal{M}_{P,s}|$. The assumption is thus that candidate nomination at the state level is a highly noisy process. As I argue in section 4.1, this seems realistic. Each winner of a state election then becomes a member of the candidate pool of their party for the federal election. Denote by $\mathcal{M}_{P,f}$ the set of potential candidates of party P for the federal election. The probability that a politician with platform p who belongs to $\mathcal{M}_{P,f}$ is nominated is given by a function $\eta_P(p|\mathcal{M}_{P,f})$. A possible interpretation is that the shape of the η -functions is a reflection of the mechanisms that parties have adopted for candidate selection, such as voting by party delegates, primaries, or caucuses. It might seem that the most natural candidate for this function would be the one that corresponds to the situation where the party always selects the politician closest to the federal median voter. As will become clear below, this might cause defections by politicians who stand little chance of being nominated for the federal election in this situation. The party thus has an incentive to ensure that such politicians are nominated with sufficiently high probability. It would be interesting to make the nomination mechanism a strategic choice, but this is beyond the scope of this paper.

The policy that is implemented in a state is equal to the platform of the politician elected in the state election, just like the policy at the federal level is equal to the platform of the politician elected in the federal election. The winner of each election is the candidate that achieves the highest number of votes with ties resolved randomly.⁵

2.2 Information

A crucial feature of the concept of political parties employed here is that voters have limited information about politicians. At the beginning of the game, the electorate cannot distinguish between different politicians and only knows how

⁵Even in a system of FPTP elections the implementation of policies requires a majority in parliament. With more than two parties competing the choice of policy may therefore require a process of coalition formation. I abstract from such issues here. At least the two-party equilibria presented below do not depend on what is assumed about the process of policy formation when no party achieves a majority. This is because voters will be allowed to vote strategically, which implies that there always exists a voting equilibrium with one party winning with a strict majority, even off the equilibrium path when a third party has entered.

their platforms are distributed. As there is one politician for each possible platform in each state, the prior belief of voters over the platform of a randomly selected politician thus assigns probability $|\mathcal{T}|^{-1}$ to each platform. Furthermore, voters can see which parties have nominated a candidate in their state, but not how many politicians have joined each party. Voters do know, however, how candidates are selected. This knowledge together with a belief about which politicians have joined a particular party allows voters to update their beliefs about the platform of a party's candidate prior to casting their vote at the state-level election. Suppose, for example, that a politician in a certain state is a member of a party of shape $[0.5, 1]$ and voters believe that only a politician with platform 1 joins this party. Voters will consequently believe that the candidate of the party must have platform 1. If the electorate instead believes that two politicians with platforms 0.5 and 1 have joined, they are aware that either of these will be nominated with probability one-half. Accordingly their belief over the platform of the candidate of the party will assign probability one-half to each of the platforms 0.5 and 1.

The winner of a state election implements her platform at the state level, thus revealing it to voters. Voters accordingly have full information about candidates at the federal level. All agents are also fully informed about the distribution of voters in all states and at the federal level.

2.3 Equilibrium

Given that the game features incomplete information, the appropriate equilibrium concept is sequential equilibrium. By itself, this would entail the possibility of a huge number of equilibria that exist when voters are allowed to vote strategically. I impose only one restriction: if a candidate is the unique most preferred option of a strict majority of voters, then a voting equilibrium where this candidate wins the election is selected. While such an equilibrium always exists under the stated conditions, there are typically additional equilibria where a different candidate gets elected. It nevertheless seems likely that voters will be able to solve the coordination problem in this case.

I focus on pure strategy equilibria. The following definition summarises the equilibrium concept:

Definition 1. *A party-formation equilibrium is a sequential equilibrium in pure strategies of the party-formation game that satisfies the following condition: If a candidate in some election is the unique most preferred option of a strict*

majority of voters, then this candidate wins the election.

Equilibrium objects are indicated by stars. In particular, \mathcal{P}^* will denote the set of active parties in equilibrium, while $N^* \equiv |\mathcal{P}^*|$. The expected platform of the candidate of party P in state s in equilibrium is given by $\bar{p}_{P,s}^*$. Conditional on a particular equilibrium it is also possible to compute the unconditional probability that a politician with platform p belonging to party P will be nominated for the federal election:

$$\eta_P^*(p) = E_{\mathcal{M}_{P,f}^*} [\eta_P(p|\mathcal{M}_{P,f}^*)] .$$

Finally, denote by $\omega_{P,f}^*(p)$ the probability that party P wins the federal election in equilibrium if it nominates a politician with platform p .

2.4 Voter Distributions

A crucial input of the model is the set of voters. My assumptions in this regards basically amount to assuming a minimum amount of heterogeneity in voter tastes. Figure 1 seems to indicate that actual heterogeneity across US states is substantial. Assuming the existence of some states with relatively extreme voter preferences increases the incentives of politicians to join parties targeted at particular states and therefore ensures that the model provides a non-trivial answer to the question of why such parties fail to compete successfully in reality.

I assume that the set of voters in any state s can be described by a measure V_s that assigns to any subset of \mathbb{R} the number of voters whose ideal policy lies in this subset. Let m_s denote the ideal policy of the median voter of state s . Similarly, let V_f be the measure of voters participating in the federal election with median m_f where

$$V_f(D) = \sum_{s=1}^S V_s(D)$$

for any $D \subseteq \mathbb{R}$. It is assumed that m_f is equal to zero. It will often be important to know what share of voters in some region $r \in \{1, \dots, S, f\}$ is located in some interval $[a, b]$. I will therefore define

$$\Lambda_r([a, b]) \equiv \frac{V_r([a, b])}{V_r(\mathbb{R})} .$$

Apart from the normalisation $m_f = 0$ introduced in the previous paragraph, the only assumptions imposed on voter preferences specify that there is some

minimum amount of heterogeneity in voter distributions across states: let there be at least one state s such that $m_s \leq -1$, at least one state s' such that $m_{s'} = 0$ and $\Lambda_{s'}((-0.5, 0.5)) > 0.5$, and at least one state s'' such that $m_{s''} \geq 1$. As the labels of states are arbitrary it is without loss of generality to denote these states as states 1, 2, and 3, respectively.

To fit the model more closely to the particular case of the US, it would also be possible to introduce an electoral college at the federal level. In this case the results go through unchanged if the median voter of the state with the median electoral vote is assumed to be located at zero.⁶

3 Results

The central insight of this paper is to show how the number of parties in a system of FPTP elections depends on the career concerns of politicians. In particular, the main result demonstrates that two parties jointly dominating elections across all levels of government is a possible outcome only if politicians care sufficiently strongly about winning elections at the federal level. To gain some intuition for why this is the case, note that politicians are generally happier the fewer members their party has: a higher number of members entails more competition for the party nomination. Suppose for a moment that all that politicians cared about was being elected at the state level. In this case, if any politicians had a chance of joining a party with fewer members that nevertheless allows them to win the state election, surely they would take it. An equilibrium where only two parties attract members could then exist only if these parties are positioned in a way that makes it impossible for third-party candidates to win any elections. Two features of the model aid parties in doing so: First of all, coordination failure among voters was not ruled out and this can make entry of third parties difficult. Secondly, the types of politicians who join a party are not necessarily the same across all states. This enables parties to have a different ideological profile in different states (albeit only to a limited extent, as will become clear below). Nevertheless, Proposition 1 will demonstrate that in any two-party equilibrium there are politicians who could compete successfully at the state level after joining a smaller, more ideologically-targeted third party.

⁶The median electoral vote can be calculated as follows: Create a distribution of electoral votes by taking the median voter among the general electorate of each state and assigning to it the electoral college votes of the state. Then find the median of this distribution. When there are two parties competing at the federal election, the party closest to the median voter of the state with the median electoral vote wins a majority of electoral votes.

As a consequence, any two-party equilibrium ceases to exist as the payoff from winning a state election becomes large relative to the payoff of winning the federal election (Proposition 2).

In deriving the results, it will be useful to be able to state concisely that a politician has the opportunity to win a state election by joining a previously passive party. I will say that such a politician can contest the state election. More formally, denote by $\mathcal{M}_{P,s}^*$ the membership of party P in state s in an equilibrium where the set of parties \mathcal{P}^* has attracted members. A politician with platform p in state s can contest the state election if it is the case that

$$u(|p - i|) > \sum_{p' \in \mathcal{M}_{P,s}^*/p} \frac{1}{|\mathcal{M}_{P,s}^*/p|} u(|p' - i|) \quad \forall P \in \mathcal{P}^* \quad (1)$$

is satisfied for a strict majority of voters. In this case, if politician p were to deviate and join a party P' with shape $[p]$, she would win the state election.⁷ In addition, I will say that a state is contestable in an equilibrium if there exists a politician who can contest it.

I will now present two lemmas that partially characterize equilibrium behaviour. The first one gives some necessary conditions for the behaviour of politicians to be consistent with equilibrium. Most importantly, if a party wins a state election, then all eligible politicians must have joined the party in this state in any two-party equilibrium.

Lemma 1. *Consider any equilibrium. Then any politician who is eligible to join a party that wins at least one state election must have done so, while all other parties attract no members. Furthermore, if $N^* = 2$, then any politician in any state s who is eligible to join the party that wins the election in state s must have done so.*

An implication of the preceding lemma is that a party that is successful in a state will always feature internal competition for the party nomination in that state. The only exception is when a party allows only one type of politician to join. However, when there are only two parties at least one of them must allow more than one type politician to join; otherwise the heterogeneity in voter preferences across states would allow a third party to enter successfully. This is shown by the following lemma:

⁷Lemma 3 in the appendix implies that in this case voters would hold correct beliefs about which parties politicians have joined and accordingly equation (1) is appropriate to determine that a voter strictly prefers the candidate of party P' over any other candidate.

Lemma 2. *In any equilibrium it cannot be the case that $I_P \cap I = \emptyset$ for all $P \in \mathcal{P}^*$ and for any $I \in \{-2, -1\}, (-1, 1), [1, 2]\}$.*

Combined, Lemmas 1 and 2 imply that in any two-party equilibrium there must be at least one party where members are competing for the nomination at the state level and each one of them is thus less than certain to win the state election. A politician who can contest a state election by joining a previously passive party with less internal competition may thus be tempted to do so. As it turns out, in any two-party equilibrium there are at least some politicians who have this opportunity.

Proposition 1. *In any equilibrium such that $N^* = 2$ there exists at least one state that is contestable.*

Parties do not necessarily have the same set of members across all states, even in equilibrium. As the proof of Proposition 1 shows, the heterogeneity in voter distributions across states nevertheless makes it impossible for two parties to appeal to all state electorates to the same extent. There will thus always be some politicians in any two-party equilibrium who could deviate and be successful at the state level as a member of a third party. If these politicians care strongly about being elected at the state level relative to career opportunities at the federal level this deviation will be profitable and any constellation of two parties is not stable. This is demonstrated by the following proposition.

Proposition 2. *For any constellation of two parties \mathcal{P}_2 , there exists a constant $\bar{y} > 0$ such that an equilibrium in which $\mathcal{P}^* = \mathcal{P}_2$ exists only if $y_f/y_s \geq \bar{y}$.*

The previous result shows that two-party equilibria can only exist if politicians care sufficiently strongly about career prospects at the federal level. However, it still remains to be shown that a two-party equilibrium exists at all. I will therefore now derive sufficient conditions for the existence of an equilibrium in which two parties, L and R , are formed along the equilibrium path, where party L allows politicians located to the left of the federal median voter to join, while party R admits only politicians to the right of the federal median voter. This constellation of parties closely resembles that observable in the US.

Constructing such an equilibrium first of all requires that parties need to extend their membership far enough to the extremes such that any politician who can contest a state election is able to join a party. Otherwise an additional party would certainly enter. For any state s such that $m_s \leq 0$, denote by $p_{L,s}$

the smallest $x \in \mathcal{T}$ such that

$$u(|x - m_s|) \geq \sum_{p \in (x, 0] \cap \mathcal{T}} \frac{1}{|(x, 0] \cap \mathcal{T}|} u(|p - m_s|) .$$

That is, $p_{L,s}$ denotes the platform of the most left-wing politician that is preferred by the median voter of state s to the nominee of party L in case that party L allows all politicians between $p_{L,s}$ and 0 to join. As u is decreasing, it must be true that $p_{L,s}$ falls in between $2m_s$ and the smallest element of \mathcal{T} that is larger than m_s . Define

$$p_L \equiv \min_{s \text{ s.t. } m_s \leq 0} p_{L,s} .$$

If party L has the shape $[p_L, 0]$, no party located to the left of party L can enter and successfully contest a state election. Similarly, for any state s such that $m_s \geq 0$, let $p_{R,s}$ equal the largest $x \in \mathcal{T}$ such that

$$u(|x - m_s|) \geq \sum_{p \in [0, x) \cap \mathcal{T}} \frac{1}{|[0, x) \cap \mathcal{T}|} u(|p - m_s|)$$

and define

$$p_R \equiv \max_{s \text{ s.t. } m_s \geq 0} p_{R,s} .$$

Proposition 3. *An equilibrium of the party formation game where $\mathcal{P}^* = \{L, R\}$, with $I_L = [p_L, 0]$ and $I_R = [0, p_R]$, exists if for any politician p in some state s who joins party $P \in \{L, R\}$ in equilibrium the following conditions are satisfied:*

i) If party P wins in state s and politician p can contest the election then

$$\Lambda_f \left(\left[\frac{p_L + p}{2}, \frac{p + p_R}{2} \right] \right) \leq 0.5 .$$

ii) If party P wins in state s and politician p can contest the election then

$$\left(1 - \frac{1}{|\mathcal{M}_{P,s}|} \right) y_s \leq \frac{\eta_P^*(p) \omega_{P,f}^*(p)}{|\mathcal{M}_{P,s}|} y_f .$$

iii) If party P does not win the election in state s then politician p cannot contest the election.

Conditions ii) and iii) are also necessary for the existence of such an equilib-

rium.

I will refer to the equilibrium in the preceding proposition as the L - R equilibrium. What prevents additional parties from forming in this equilibrium? After all, there will always be politicians in any two-party equilibrium who could successfully contest a state election as a member of a third party, as was shown above. But while this is true, the potential for success of third parties does not extend to the federal level. As parties L and R are positioned symmetrically around the federal median voter, third-party candidates have little chance of winning the federal election. In fact, condition i) of Proposition 3 ensures that any third-party candidate loses the federal election. The only benefit that newly formed parties can offer politicians then is that they may enable politicians to win the state election with higher probability due to lower internal competition for the party nomination. Parties L and R , on the other hand, offer career prospects at the federal level. If politicians value such opportunities sufficiently strongly—as expressed in condition ii) of Proposition 3—entrant parties are unable to attract members.

As can be seen from condition ii) of Proposition 3, the ability of parties to prevent their members from defecting depends on the nomination technology η_P used at the federal level. Parties can increase their chances of winning the federal election by nominating centrist politicians with high probability. The lower the probability that extremist politicians are nominated, however, the more likely they are to join a third party. If they do so, this would be highly problematic for the party they are leaving behind. Suppose, for example, that some members of party L defect and form a more left-wing party. This can lead to a split in the left-wing vote, handing victory at the federal election to party R . Party L would therefore prefer to grant extremist politicians a somewhat higher probability of nomination if this maintains the unity of the party.

It is noteworthy that the L - R -equilibrium is able to reproduce the pattern in US election results presented in the introduction: States with extreme median voters will display large majorities in favour of one of the parties, while in states with median voters close to zero the margin of victory will be small. In addition, the federal election will be competitive, particularly if neither party extends much further to the extremes than the other and both of them use a similar nomination technology for the federal election. The model thus not only provides an explanation for the absence of successful third parties in the US, but is also able to match empirical patterns.

I will conclude this section by discussing other types of equilibria. If career concerns are not sufficiently strong and the L - R equilibrium does not exist, other two-party equilibria might. Excluding some centrist politicians from both parties would lower internal competition and make extremist politicians less likely to defect. If the gap between parties becomes too large, however, centrist politicians not affiliated to any party will become able to contest elections in centrist states, upsetting the equilibrium. On the other hand, a two-party equilibrium with stronger overlap than in the L - R equilibrium might also exist if politicians care strongly about success at the federal level. Such overlap can be hard to maintain in equilibrium though: it creates the possibility that both parties nominate politicians for the federal election who are located on the same side of the federal median voter. This would make it relatively easy for voters to coordinate on electing a third-party candidate. For example, a centrist candidate of a third party can attract all voters to left of the centre if the remaining candidates are both located on the right. But if centrist politicians can do well both in state elections and in the federal election after joining a third party, nothing can prevent them from defecting.

For similar reasons an equilibrium with only one party is very unlikely to exist. If only a single party is nominating a candidate for the federal election, there is effectively no coordination problem for voters even if the candidate of a second party enters the race. Unless the equilibrium party is itself very likely to nominate a candidate with platform 0, centrist politicians can win the federal election with positive probability after joining a second party. And this probability need not be large in order to make the deviation attractive: A single party can only win all elections if it is sufficiently broad to allow even extreme politicians to join, who would otherwise be able to contest elections in states with similarly extreme median voters. As a result, there is intense internal competition for nominations. Centrist politicians are therefore likely to be better off as members of a different party. This is formalised by the following proposition.

Proposition 4. *There is no equilibrium such that $N^* = 1$ if any member of the candidate pool for the federal election of the party active in equilibrium with a platform other than zero is nominated with a probability of at least*

$$\frac{1}{(S-1)(|\mathcal{T} \cap [-1, 1]| - 1)}$$

on average.

According to Proposition 4, the likelihood that a one-party equilibrium exists is decreasing both in the number of states and the number of politicians located in the interval $[-1, 1]$. The higher the number of states, the higher the number of state winners vying for the federal nomination of the equilibrium party. As these are likely to have a platform not equal to 0, this makes it easier for centrist candidates of another party to win the federal election. A higher number of politicians located in the interval $[-1, 1]$ has the same effect, as it decreases the expected number of politicians with platform 0 in the candidate pool of the equilibrium party. The case in which a one-party equilibrium is most likely to exist is therefore when the number of states is equal to four and only three platforms fall into the interval $[-1, 1]$. In this case a politician with platform 0 who has won a state election can be nominated for the federal election with a probability as high as two thirds ex-ante and would still prefer to deviate and join a second party. A one-party equilibrium is therefore highly unlikely to exist. This final result is noteworthy as there are no democratic countries where one party dominates all levels of government.⁸

4 Robustness

The basic model of party formation presented above requires a number of simplifying assumptions for tractability. This section will discuss some of these in more detail.

4.1 Candidate Selection and Mixed Strategies

It was assumed that any member of a party in a state is nominated with equal probability for the state election. Generally, I feel that assuming that candidate selection at the state level is a fairly noisy process is realistic. The potential candidates are largely unknown to voters, making it difficult for parties to commit to nominating a particular type of politician. Furthermore, different factions within the party will be in disagreement about the ideal candidate. While the party leadership at the national level has an interest in supporting moderates who will later on make suitable candidates for federal offices, party activists within a state will likely be pushing for more extreme nominees. Furthermore,

⁸It should be clear from the discussion above that the existence of a one-party equilibrium depends on the career concerns of politicians just as in the case of equilibria with two parties.

ignoring parts of the party membership when deciding the nomination will likely lead to defections.

It therefore seems doubtful that parties will be able to target their candidates with great precision at the median voters of different states. And small differences in the expected platforms of candidates of the same party in different states are unlikely to upset the results. While this would make it easier for two parties to fend off entry of a third one, other assumptions already favour incumbents over entrants. In particular, the bar for voters to coordinate on electing a third-party candidate was set relatively high. In addition, I only assume a minimum amount of heterogeneity in voter tastes across states. A small increase in the ability of parties to differentiate could be countered by a small increase in the assumed amount of heterogeneity and Proposition 1 survives.

Mixed strategy equilibria may also enable parties to have a different ideological profile in different states and the same arguments apply as in the previous paragraph. Furthermore, mixed strategies can only have a substantial impact on the expected platform of candidates in cases where parties overlap strongly. While this is difficult to show in general, it seems unlikely that two broad and largely overlapping parties could preclude entry of a third party even if mixed strategies are taken into consideration.

4.2 Policy Choices

The assumption that politicians are committed to implementing their platform is not satisfying. While the empirical literature quoted above seems to suggest that policy preferences of politicians are the main driver of their choices in office, it would be more appealing to see this behaviour emerge as part of an equilibrium rather than imposing it from the outset. In the model, extremist politicians can often increase their chances of winning the federal election by pretending to be a centrist when choosing state policies. To address this concern I will briefly consider a more general utility function for politicians that includes both career concerns and policy preferences. For a politician with ideal policy i who joins a party that wins at least one election let the utility function now be given by

$$y_w + \pi_s(y_s + \pi_f y_f) - \alpha \sum_{l \in \{s, f\}} u(|p_l - i|),$$

where α measure the the relative weight that politicians attach to policy and the notation is otherwise the same as in Section 2. Parties then allow only politicians

with certain ideal policies to join. In addition, assume that politicians can freely choose the policy they implement at any stage. All other elements of the game remain unchanged. This more general version of the model is challenging to solve in its entirety. However, focusing on the subgame reached after state elections have been held, it is clear that a separating equilibrium exists where politicians implement their ideal policy at the state level if α is sufficiently large: Given that politicians behave in this way, some politicians might be able to increase their chances of being elected at the federal election by choosing a different policy after winning a state election. In an equilibrium where everyone behaves truthfully, voters will then expect this politician to implement the same policy if elected federally and might be more likely to vote for her. The gain in utility associated with this increase in the likelihood of winning the federal election is clearly finite though. If α is sufficiently large, the loss in utility associated with implementing a less-than-ideal policy at the state level will weigh more heavily and the deviation is not profitable.

Even if such a separating equilibrium does not exist, however, behaviour doesn't necessarily change drastically. The party that a politician belongs to puts limits on the ideal policy that a politician can have and therefore also on the beliefs that voters can form about this ideal policy. An equilibrium in this case might see all winners of state elections pool on the most centrist policy that a member of their party is allowed to pursue. In some cases, such as in the *L-R* equilibrium presented above, this can mean that all politicians chose the same policy at the state level (the policy 0 in the case of the *L-R* equilibrium). Voters would then nevertheless benefit from voting for parties positioned closely to them as this can pay off if one of their members gets elected at the federal election.

5 Conclusion

Why are the same two parties competing in elections in the US across all levels of government, while parties in other countries relying on FPTP elections are much less integrated between the national and the regional level? In this paper I have highlighted the career concerns of politicians as a possible explanation. In my model, heterogeneity in voter tastes across states forces two parties to adopt a broad ideological profile. This leads to intense internal competition for nominations. Joining a smaller party more targeted at the preferences of voters

in a particular state would allow some politicians to win the state election with higher probability. The drawback of this move is that politicians miss out on the career opportunities that federally successful parties offer. As a consequence, two-party equilibria exist only if politicians value such opportunities sufficiently strongly.

In addition, I provide existence conditions for a particular equilibrium with two parties that looks very similar to what we observe in the US. In this equilibrium both parties allow centrist politicians to join, while one party extends its membership far enough to the left to prevent entry of a left-wing party and the other party does the same on the opposite end of the political spectrum. As a consequence, this equilibrium reproduces the pattern in election results presented in Figure 1 in the introduction.

While this paper has focused on FPTP elections, a similar logic should also apply to countries using proportional representation, such as Austria or Germany. Both countries feature two main parties that traditionally (if less so recently) receive the vast majority of votes. Importantly, this is true federally as well as at the state level. It thus seems that the major parties allow for and attract a membership that is ideologically broad enough to ensure a strong position across states. Preventing entry of any additional parties, in contrast, would require extending the party membership far to the extremes under proportional representation and could be too costly in terms of votes lost at the federal level and in more moderate states. Career concerns would then again be an important factor in that they prevent fringe parties from luring politicians away from the major parties. The technical difficulties involved in modelling systems of proportional representation make this a challenging subject for further research.

Appendix: Proofs

The following two lemmas are not presented in the text:

Lemma 3. *Consider an equilibrium of the party-formation game and suppose that a politician with platform p in some state s deviates and joins a party P with shape $I_P = [p]$. Then voters' belief at the information set reached must place full probability on politician p in state s having joined party P while the behaviour of all other politicians has not changed.*

Proof. Index politicians by $j \in \mathcal{J} = \{1, \dots, S \cdot |\mathcal{T}|\}$ and denote by p_j the platform of politician j . Let $\mathcal{N}(j)$ be a node of some information set $\tilde{\mathcal{N}}$ reached after politicians have made their affiliation choices, that is $\mathcal{N}(j)$ assigns one element of $\mathcal{I}(p_j) \cup \{\emptyset\}$ to each politician j . Let $\sigma_j : \mathcal{P}(p_j) \cup \{\emptyset\} \rightarrow [0, 1]$ describe a strategy of a politician while σ is a vector of strategies for all politicians.

Let \mathcal{N}^* be the node of information set $\tilde{\mathcal{N}}^*$ reached along the equilibrium path of some pure-strategy equilibrium where each politician j uses strategy σ_j^* . Then sequential equilibrium requires that

$$\lim_{n \rightarrow \infty} \frac{\prod_{j \in \mathcal{J}} \sigma_j^n(\mathcal{N}^*(j))}{\sum_{\mathcal{N} \in \tilde{\mathcal{N}}^*} \prod_{j \in \mathcal{J}} \sigma_j^n(\mathcal{N}(j))} = 1 \quad (2)$$

for some sequence σ^n of totally mixed strategies such that $\sigma^n \rightarrow \sigma^*$. The requirement that $\sigma^n \rightarrow \sigma^*$ implies that the numerator and denominator in equation (2) must both approach one as n gets large. Furthermore,

$$\lim_{n \rightarrow \infty} \prod_{j \in \mathcal{J}/k} \sigma_j^n(\mathcal{N}^*(j)) = 1 \quad \forall k \in \mathcal{J}. \quad (3)$$

Now suppose that some politician k in some state s deviates from σ_k^* and joins a party P' with shape $[p_k]$. Voters then observe that party P' starts campaigning in state s . Let $\tilde{\mathcal{N}}'$ be the information set reached after the deviation. As politician k is the only politician who can join party P' in state s , $\mathcal{N}'(k) = P'$ for all $\mathcal{N}' \in \tilde{\mathcal{N}}'$. The belief that some node \mathcal{N}' of information set $\tilde{\mathcal{N}}'$ has been reached in the sequential equilibrium under consideration can therefore be written as

$$\lim_{n \rightarrow \infty} \frac{\prod_{j \in \mathcal{J}/k} \sigma_j^n(\mathcal{N}'(j))}{\sum_{\mathcal{N}' \in \tilde{\mathcal{N}}'} \prod_{j \in \mathcal{J}/k} \sigma_j^n(\mathcal{N}'(j))}. \quad (4)$$

Now let \mathcal{N}' be the node actually reached after the deviation of politician k . Then it is true that $\mathcal{N}'(j) = \mathcal{N}^*(j)$ for all $j \neq k$. This, together with equation (3), implies

$$\lim_{n \rightarrow \infty} \prod_{j \in \mathcal{J}/k} \sigma_j^n(\mathcal{N}'(j)) = \lim_{n \rightarrow \infty} \prod_{j \in \mathcal{J}/k} \sigma_j^n(\mathcal{N}^*(j)) = 1,$$

which shows that the limit in expression (4) must also be equal to one in this case. \square

Lemma 4. *In any equilibrium*

- i) *state 1 is contestable unless there exists at least one party P such that $\bar{p}_{P,1}^* \leq -1$,*
- ii) *state 2 is contestable unless there exists at least one party P such that $\bar{p}_{P,2}^* \in (-1, 1)$, and*
- iii) *state 3 is contestable unless there exists at least one party P such that $\bar{p}_{P,3}^* \geq 1$.*

Proof. It will be shown that state 1 is contestable if $\bar{p}_{P,1}^* > -1$ for all $P \in \mathcal{P}^*$. Assume this condition was satisfied and suppose the politician with platform -1 in state 1 joins some party D with shape $[-1]$. According to Lemma 3, voters must then believe that politician -1 has deviated while all other players still follow their equilibrium strategies. But then the expected platform of the candidate of party D is equal to -1 with certainty while the expected platforms of all other parties' candidates must be greater than -1 even after the deviation. Accordingly the median voter and all voters located further to the left must strictly prefer the candidate of party D and party D wins the state election. Similarly, state 2 is contestable by a politician with platform 0 if $\bar{p}_{P,2}^* \notin (-1, 1)$ for all $P \in \mathcal{P}^*$ as $\Lambda_2((-0.5, 0.5)) > 0.5$, while state 3 is contestable by a politician with platform 1 if $\bar{p}_{P,3}^* < 1$ for all $P \in \mathcal{P}^*$. \square

Proof of Lemma 1. Suppose there was an equilibrium such that some party P wins at least one election and that there is a politician with platform p in state s such that $p \in I_P$ who does not join a party in equilibrium. This politician therefore receives a payoff of zero. If the politician joins party P instead, she receives a positive payoff unless party P wins no elections other than in state s in equilibrium and loses even this election after the deviation. But if party P wins the election in state s in equilibrium then it must also have some members in state s in equilibrium. As voters only observe whether a party is campaigning at the state level, but not how many politicians have joined, voters therefore do not detect that a deviation has taken place and party P continues to win the state election. Hence, the deviation is profitable. On the other hand, any politician who joins a party that does not win any elections receives a negative payoff by assumption and would prefer not to join a party.

To show the final part of the statement, assume that some party P wins the election in some state s and there is a politician with platform p in state s such that $p \in I_P$ who does not join party P in equilibrium. If the politician joins party P , voters can only observe that a deviation has taken place if the politician was a member of a different party before and this party has no other members in the state after the deviation. But in any two-party equilibrium this would imply that party P is the only remaining party in the state after the deviation, as it was argued above that parties that do not win elections have no members in equilibrium. Party P will therefore always continue

to win the state election after the deviation and as each member of a party at the state level is nominated with positive probability, the deviation is profitable, contradicting equilibrium. \square

Proof of Lemma 2. Suppose $I_P \cap (-1, 1) = \emptyset$ for all $P \in \mathcal{P}^*$. The expected platform of any nominee therefore lies outside of $(-1, 1)$ and Lemma 4 accordingly implies that politician 0 in state 2 is able to contest the state election. But $I_P \cap (-1, 1) = \emptyset$ for all $P \in \mathcal{P}^*$ requires that politician 0 in state 2 has not joined a party in equilibrium and this politician therefore has a profitable deviation. The remaining cases can be shown analogously. \square

Proof of Proposition 1. Consider an equilibrium such that $\mathcal{P}^* = \{A, B\}$. Denote by \mathcal{M}_P the set of politicians in any given state that is allowed to join party $P \in \{A, B\}$ and let $\mathcal{M}_{P \setminus -P} = \mathcal{M}_P \setminus \mathcal{M}_{-P}$ with $P \in \{A, B\}$ and $-P \in \{A, B\} \setminus P$. Let $\bar{p}_P(\mathcal{M})$ be the expected candidate of party P in any state where the set of politicians \mathcal{M} has joined party P , that is

$$\bar{p}_P(\mathcal{M}) \equiv \sum_{p \in \mathcal{M}} \frac{1}{|\mathcal{M}|} p .$$

Now assume that in equilibrium no state was contestable. By Lemma 1, it must be true in equilibrium that

$$\bar{p}_{P,s}^* \in \{\bar{p}_P(\mathcal{M}_P), \bar{p}_P(\mathcal{M}_{P \setminus -P})\} \quad \forall s \in \{1, \dots, S\}, P \in \{A, B\} . \quad (5)$$

The two parties combined can thus generate up to four different expected platforms. According to Lemma 4, one of these must lie weakly below -1, one in the interval $(-1, 1)$, and one weakly above 1. Without loss of generality, assume that $\bar{p}_A(\mathcal{M}_A) \leq \bar{p}_B(\mathcal{M}_B)$. Together with the fact that candidates are nominated uniformly at random this implies

$$\bar{p}_A(\mathcal{M}_{A \setminus B}) \leq \bar{p}_A(\mathcal{M}_A) \leq \bar{p}_B(\mathcal{M}_B) \leq \bar{p}_B(\mathcal{M}_{B \setminus A}) , \quad (6)$$

as long as all of these quantities are defined.⁹ The restrictions in (6) combined with the requirements of Lemma 4 immediately rule out a number of possible cases. For example, if party A wins in state 1 and $\bar{p}_A(\mathcal{M}_A) \leq -1$ and party B wins in state 3 and $\bar{p}_B(\mathcal{M}_B) \geq 1$, then condition (6) implies that there can be no candidate in state 2 whose platform lies in the interval $(-1, 1)$, as required by Lemma 4. Three possible cases are less straightforward to rule out: *i*) party A wins in state 1 and state 3 and $\bar{p}_A(\mathcal{M}_A) < -1$ and $\bar{p}_B(\mathcal{M}_{B \setminus A}) > 1$, *ii*) party B wins in state 1 and

⁹As the expected platform of party A with full membership is weakly smaller than the expected platform of party B with full membership, removing anyone who is able to join party B from party A must remove more members above the expected platform of party A (if any). Thus it must hold that $\bar{p}_A(\mathcal{M}_{A \setminus B}) \leq \bar{p}_A(\mathcal{M}_A)$.

$\bar{p}_A(\mathcal{M}_{A \setminus B}) < -1$ while party A wins in state 3 and $\bar{p}_B(\mathcal{M}_{B \setminus A}) > 1$, or *iii*) party B wins in state 1 and state 3 and $\bar{p}_A(\mathcal{M}_{A \setminus B}) < -1$ and $\bar{p}_B(\mathcal{M}_B) > 1$. It will be shown that all of these constellations are impossible. Consider case *i*). As candidates are selected uniformly at random, $\bar{p}_A(\mathcal{M}_A) < -1$ requires $I_A \subseteq [-2, 0]$. Similarly, it must be true that $\mathcal{M}_{B \setminus A} \subseteq [0, 2]$. But then any possible candidate of party B in state 3 is located closer to the median voter of state 3 than any possible candidate of party A , contradicting that party A wins the state election in equilibrium. Next consider case *ii*). $\bar{p}_A(\mathcal{M}_{A \setminus B})$ and $\bar{p}_B(\mathcal{M}_{B \setminus A})$ both exist only if neither party is a subset of the other. $\bar{p}_A(\mathcal{M}_{A \setminus B}) < -1$ requires $\mathcal{M}_{A \setminus B} \subseteq [-2, 0]$. The utility of the median voter of state 1 if party A wins the state election must therefore be greater than $u(|0 - m_1|)$ as $m_1 \leq -1$. The median voter can then only prefer party B with full membership if $\bar{p}_B(\mathcal{M}_B) < 0$ as voters are risk averse. Analogously it must be true that $\bar{p}_A(\mathcal{M}_A) > 0$. But this contradicts the assumption that $\bar{p}_A(\mathcal{M}_A) \leq \bar{p}_B(\mathcal{M}_B)$. Finally, case *iii*) can be shown to yield a contradiction analogously to case *i*). \square

Proof of Proposition 2. Consider an equilibrium such that $\mathcal{P}^* = \mathcal{P}_2$ and suppose there is a politician in a state s with platform p who is able to contest a state election. Suppose that in equilibrium politician p is a member of party A . If party A allows more than one type of politician to join then the equilibrium payoff of politician p is bounded from above by

$$y_w + \frac{1}{2}(y_s + y_f) .$$

If politician p deviates and joins a party D that allows only politicians with platform p to join, then she achieves at least a payoff of $y_w + y_s$ as it was assumed that the politician can contest the state election. If $y_s > y_f$, then politician p in state s strictly prefers joining party D , contradicting equilibrium.

As at least one state is contestable in any two-party equilibrium by Proposition 1, the only possibility of a two-party equilibrium that exists for any value of $y_f/y_s > 0$ would then be that one of the parties allows only one type of politician to join. Suppose such an equilibrium existed and denote the parties that attract members in equilibrium as A and B . Let A be the party that allows only one type of politician to join. By Lemma 2, party B must then allow more than one type of politician to join. It will be shown that at least one politician who is not a member of party A can contest a state election. Suppose $I_A \subset [-2, -1]$. Then state 3 is contestable unless $\bar{p}_{B,3}^* > 1$ by Lemma 4. This requires $I_B \subseteq [0, 2]$. But in this case parties A and B do not overlap, so $\bar{p}_{B,s}^* > 1$ must hold across all states, and accordingly the politician with platform zero in state 2 must be able to contest the state election, again using Lemma 4. The case $I_A \subset [1, 2]$ can be ruled out analogously. The remaining case to consider is $I_A \subset (-1, 1)$. In this case Lemma 4 requires that either $\bar{p}_B(\mathcal{M}_B) < -1$ and $\bar{p}_B(\mathcal{M}_{B \setminus A}) > 1$ or the reverse inequalities hold (using the notation defined in the proof of Proposition 1). But as \mathcal{M}_B and $\mathcal{M}_{B \setminus A}$ differ by only one politician with a

platform $p \in (-1, 1)$, this is impossible. \square

Proof of Proposition 3. Politicians with platform 0 are the only politicians who can join both party L and party R . Lemma 1 implies that these politicians must have joined the party that wins the election in their state. It needs to be established that no politician has an incentive to join any parties other than L and R . Politicians who are unable to join any of the equilibrium parties cannot win the election in their state by joining an additional party. This is true by construction as parties extend from p_L to p_R . Politicians who can join party L or party R but are unable to win the state election if nominated would similarly join any party that enables them to contest the state election. Consequently, this must be impossible, making condition *iii*) a necessary condition. Finally, consider politicians whose party wins the state election in their state and who can successfully contest the state election by joining some third party D . As such a politician is the only member of party D at the point of deviation, she will be nominated for the federal election with certainty and face competing candidates from parties L and R . If condition *i*) is satisfied, there exists a voting equilibrium such that the candidate of party D loses the federal election for any constellation of the three candidates. The highest payoff a politician can achieve by deviating to joining party D is therefore $y_w + y_s$. The equilibrium payoff of such a politician who is a member of party $P \in \{A, B\}$ on the other hand is

$$y_w + \frac{1}{|\mathcal{M}_{P,s}^*|} [y_s + \eta_P^*(p) \omega_{P,f}^*(p) y_f] .$$

The inequality that states that the equilibrium payoff must be at least as large as the deviation payoff is equivalent to condition *ii*). If a politician can contest a state election, $y_w + y_s$ is also the lowest possible payoff that this politician can achieve by deviating, implying that condition *ii*) is a necessary condition for equilibrium existence. \square

Proof of Proposition 4. Suppose one party wins all elections and call this party A . By Lemma 2, $[-1, 1] \subset I_A$ and combined with Lemma 1 this implies that party A has at least $|\mathcal{T} \cap [-1, 1]|$ members in any state while all other parties have no members. A politician with platform 0 therefore achieves a payoff in equilibrium that is no greater than

$$y_w + \frac{1}{|\mathcal{T} \cap [-1, 1]|} (y_s + y_f) . \quad (7)$$

Suppose the politician with platform 0 deviates and becomes a member of a party of shape $[0]$. Refer to this party as party D . If the politician with platform 0 joins party D in state 2, voters observe that party D starts campaigning in the state and therefore know that the only eligible politician must have joined while all other politicians have not changed their behaviour by Lemma 3. As the expected platform of the candidate

of party D is then equal to the ideal policy of the state median voter with certainty while the platform of the candidate of party A has positive variance, party D must win the state election. Politician 0 from state 2 is then nominated with certainty as party D 's candidate for the federal election. As only two parties are competing in the federal election, party D must win with certainty whenever party A nominates a candidate with platform unequal to 0, which is the federal median voter's ideal policy. As candidates are selected uniformly at random at the state level, any member of party A who wins a state election has a platform other than 0 with probability at least equal to $1 - 1/|\mathcal{T} \cap [-1, 1]|$. If politicians with a platform other than 0 are nominated by party A with an average probability of $\eta_A(-0)$ ex-ante, the probability that party A nominates such a politician after the deviation by politician 0 in state 2 is at least equal to $(S - 1)(1 - 1/|\mathcal{T} \cap [-1, 1]|) \eta_A(-0)$. The payoff of politician 0 in state 2 from the deviation is thus at least equal to

$$y_w + y_s + (S - 1) \left(1 - \frac{1}{|\mathcal{T} \cap [-1, 1]|} \right) \eta_A(-0) y_f ,$$

which is greater than expression (7) for any possible values of y_s and y_f if

$$(S - 1) \left(1 - \frac{1}{|\mathcal{T} \cap [-1, 1]|} \right) \eta_A(-0) \geq \frac{1}{|\mathcal{T} \cap [-1, 1]|} .$$

This inequality is equivalent to the condition in the statement of the proposition. \square

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