Competing Candidates, Competing Interest Groups, and the Efficacy of Political Threats

Nicolas Motz*

Department of Economics, University College London (UCL)

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Abstract

Interest groups seem to achieve large policy favours for small sums of campaign contributions. This has long puzzled observers. I provide an explanation of this so called "Tullock paradox" that is robust to competition among opposing interests. In the model, I allow interest groups to specify their donations as very general functions of policies and donations by other groups. This allows potential donors to influence the policy choice of an incumbent through threats of contributions to the campaign of a challenger. It is therefore possible that the incumbent chooses policies that favour a particular interest group even if this group has not made any actual donations. When lobbies face a small amount of uncertainty about the policy that the incumbent will choose, I am able to provide a clear characterisation of equilibrium. Policies are always skewed in favour of the group with deeper pockets. This group may also use actual donations on top of threats in order to increase its influence over policies. The weaker lobby, on the other hand, does not promise any money for any policy the incumbent may implement. Outcomes nevertheless differ from the case with only one interest group as the weaker group can become active if the stronger group tries to exert even more pressure.

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1 Introduction

Political advertising is a fundamental element of any modern election campaign. Candidates use a multitude of means such as television or newspaper adverts, bill boards, or door-to-door canvassing in order to convince members of the public to vote for them. The importance that candidates attach to these activities is probably best illustrated by the share of their time devoted to raising the required funds. In the U.S., for example, a survey of former candidates found that more than 50 percent of those running for statewide office spent more than a quarter of their time eliciting campaign money. 23 percent of candidates even reported that such activities took up more than half of their time (Herrnson & Faucheux 2000). U.S. presidents attend fund-raisers not only during their own campaigns, but even during their second and final term in office. Former president Bill Clinton attended 471 such events during his second term, nearly three times more than during his first four years in office (Doherty 2013). Such observations have raised concerns that the need to finance campaigns has significant opportunity costs in terms of less time spent fulfilling official duties. Even more worrying to some is the possibility that their high demand for campaign funds makes politicians willing to trade policy favours in return for donations.

It has been questioned, however, whether such quid pro quo is actually occurring. The argument is based on a simple observation first made by Tullock (1989) and later re-emphasised by Ansolabehere et al. (2003), now commonly referred to as the Tullock paradox: When compared to the value of government regulations and subsidies, the amount spent on lobbying efforts and contributions seems small. If these expenditures are viewed as political investments, simple back-of-the-envelope calculations reveal exorbitant rates of return. Ansolabehere et al. (2003) list a number of U.S. industries whose sum of campaign contributions is dwarfed by the gains that government policies imply for them. As a consequence, one should expect that interest groups on the losing side of the bargain would increase their own contributions in order to capture a greater share of the spoils. In other words, competition should eliminate excessive rates of return. Why does this not seem to happen? Barriers to entry could play a role. It is commonly argued that collective action problems may prevent certain groups from launching effective lobbying efforts. Given the apparent size of the benefits to be reaped, this argument seems only partially convincing.

In this paper I propose a possible explanation of the Tullock paradox. I show that it might not be necessary to make contributions in order to have influence;

the mere threat of contributions may be enough. In the model presented in the paper an incumbent is facing reelection under competition from a challenger. The incumbent is willing to trade policy favours in return for donations from an interest group. Now it might be possible for the interest group to secure the same favours, not by contributing, but simply by threatening the incumbent with a donation to the challenger. Due to the zero-sum nature of the situation, what really matters to the incumbent is not the absolute amount she spends on advertising, but by how much she out-spends the challenger. This makes threats of donations just as effective as actual contributions.

Interest groups may nevertheless give money in equilibrium. Making a contribution to the campaign of the incumbent allows for the combined threat of withdrawing this donation and simultaneously giving money to the challenger. This gives the interest group even more influence over policy choices. As these policies then reflect the contribution the incumbent receives as well as the threat she is subject to, the model generates the appearance of very high returns on actually carried out donations.

Crucially—and in contrast to the existing literature—I show that this logic remains valid even when there is more than one interest groups. As explained above, the question of why competition among interest groups does not dissipate excessive rents is at the heart of the puzzle.

In order to explain how I achieve my results I will first describe the modelling approach generally used in the literature. Since the seminal work by Bernheim & Whinston (1986), and in particular through the contributions of Grossman & Helpman (1994, 1996), it has become customary in models of campaign contributions to allow interest groups to offer schedules of donations to candidates. These schedules make the money an interest group gives to a politician conditional on the politician's policy choices or campaign promises. Importantly, these schedules are viewed as being representative of the commitment power that interest groups would have in a game of repeated elections. In other words, contracts are seen as relational rather than legal. The motivation behind introducing these contracts is that they enable interest groups to make donations with the explicit aim of influencing policy, rather than just increasing the chances of a particular candidate once campaign platform have been announced.

While the argument just given justifies the use of contribution schedules it says little about the exact nature of these contracts. As it turns out, this is crucial. In Grossman & Helpman (1996) donations are offered to a candidate as a function of this candidate's campaign platform only. This means that it is impossible to threaten candidates as their choice of platform cannot have any influence on how much money their competitor receives. In contrast, threats are possible in this paper because the policy choice of the incumbent determines how much money both she and the challenger will receive.

When two interest groups are present, the additional issue arises that each one of them might want to change its own donations in response to donations made by the other group. In fact, if contribution schedules are viewed as representing informal commitments made by lobbies in the beginning of the game, it is hard to argue why they should not have the ability of making these commitments conditional on their opponents actions[1] as well. I therefore allow interest groups to make their contributions a function of policy as well as of donations made by the other interest group. Similar contracts arise in other contexts, for example when retailers promise to match the prices of competitors, making prices a function of other prices. Peters & Szentes (2012) discuss other examples.

Independently of the particular choice of what donations can be conditioned on, the number of equilibria is large as soon as more than one interest group is present. This stems partially from the fact that interest groups are almost unconstrained in their commitments to contributions at policies that are not chosen in equilibrium. Threatening to make sufficiently high donations can ensure that these donations never actually have to be carried out. It seems desirable to introduce at least some chance that interest groups will be held to their word. I therefore require that equilibria are robust to small perturbations of the game where there is a small probability that the incumbent turns out to be an "ideologue" who sticks to some platform irrespective of how likely this choice is to lead to electoral success. Ex-ante interest groups thus perceive a small chance that they will have to carry out promises that would otherwise never be tested. This greatly reduces the complexity of possible schedules and allows me to characterize the set of equilibria more fully.

I find that the set of possible equilibrium outcomes is largely determined by the maximum amount of contributions that each interest group is able to pledge. In particular, policies are always skewed in favour of the group with deeper pockets. This lobby is also the only one that makes contributions in

[1]Note that there is a conceptual difference between making commitments conditional on observed actions of the other interest group and making commitments conditional on contribution schedules per se. The latter would require that interest groups can perfectly observe the schedule put forward by their competitor, which seems very strong to assume. Tennenholtz (2004) and Peters & Szentes (2012) pursue the idea of contracts being conditioned on other contracts further.

equilibrium and only to the incumbent. In fact, I obtain the striking result that the weaker interest group remains almost completely passive, in the sense that it does not promise any donations for any policies that the incumbent might choose. The presence of the weaker group matters for outcomes nevertheless, as the general nature of contribution schedules allows this lobby to become active if the group with deeper pockets should try to gain even more influence. Intuitively, the stronger lobby does not exert as much pressure as it potentially could, because it knows that doing so would provoke a reaction from the so-far passive group.

While the equilibrium policy is moderated by the existence of a second interest group, the stronger group still uses a combination of threats and actual contributions in order to influence policies. This generates high rates of return on donations in the same way as outlined above. Bidding wars are possible out of equilibrium, but are not initiated by the weaker group in the knowledge that they would not bring any advantage.

This paper is not the first to allow interest groups to commit to more general contribution schedules. Chamon & Kaplan (2013) present a model with two candidates who compete by announcing campaign platforms. However, they only allow for one interest group and rely on much stronger, parametric assumptions in deriving their results. This is partially due to the more applied nature of their paper, in which they also provide empirical evidence in favour of the theoretical results. As the present paper, their model predicts that split contributions—where one interest group contributes to both candidates competing in a race—should not occur, which stands in contrast to the theory of Grossman & Helpman (1996). Using data from contributions to candidates for the U.S. House of Representatives Chamon & Kaplan show that split contributions are indeed rarely observed. A second feature of their theoretical results is that candidates who win with a higher vote margin should be receiving higher contributions. Again, this is confirmed by the data. The same pattern can be generated by the model presented here. I view their work as complementary to mine.

The paper is organized as follows: In the following section I briefly discuss the related literature. Section 3 gives the details of the model. In section 4 the case with one interest group is analysed. The main results of the paper are contained in section 5, which presents the characterisation of equilibrium in the presence of two interest groups.

2 Related Literature

The idea of giving interest groups the ability to commit to contribution schedules has featured in the literature on campaign contributions ever since the introduction of the concept of a "menu auction" by Bernheim & Whinston (1986), where a number of bidders submit schedules to a seller. These schedules specify the transfers the participants in the auction will make to the seller depending on the allocation that the seller decides to implement. The main application of this theory that Bernheim & Whinston had in mind was influence seeking.

Grossman & Helpman (1994) develop such an application in the context of the design of trade policy by a single policy maker who maximizes a weighted sum of voter welfare and campaign contributions. Their model has since been widely used in both the theoretical and empirical literature on trade barriers. To justify their choice for the objective function of the policy maker in the previous paper, Grossman & Helpman (1996) analyse an election in which two candidates compete by announcing campaign platforms and spending money on advertising. Contributions to one candidate are nevertheless a function of this candidates platform only. While the authors give arguments supporting why interest groups should have the ability to make binding commitments, they do not justify the restrictions they impose on these "contracts". According to their results, interest groups make contributions to both candidates in order to influence the policies that each one of them proposes and sometimes give more to one candidate in order increase that candidate's electoral chances in particular. I show that more general contracts allow interest groups to influences campaign platforms through the mere threat of contributions.

The insight that externalities among agents (candidates in this context) can enable a principal to extract a large share of the surplus has been present in the literature on contract theory and mechanism design (Crémer & McLean 1985, Aghion & Bolton 1987, Jehiel et al. 1996, Segal 1999, Spiegler 2000). Most of these papers feature only one principal. As fare as I am aware, Spiegler (2000) is the only paper in this literature that simultaneously features both multiple agents and multiple principals.

Within the context of political economy, the literature on vote buying (Helpman & Persson 2001, Dal Bó 2007, Morgan & Várdy 2011) has identified the possibility of influencing policies without having to actually carry out any transfers. This can be achieved by promising committee members bribes in case their vote should be pivotal. As voters do not care about their vote as long as they are not pivotal, voting in favour of the interest group becomes weakly dominant for large enough promised bribes. The consequence is that all votes are cast in favour of the interest group. Consequently no single voter is pivotal and no transfers have to be carried out. While certainly related, these papers are quite different both in their context and the structure of the models they develop. In particular, legislators are not subject to threats in these papers.

3 The Model

3.1 Politicians

A politician, called the incumbent, chooses a policy p from the set $\mathcal{P} \equiv [-1, 1]$. She knows that this policy choice will have an impact on her probability of getting re-elected. She can also influence this probability by spending an amount of money $a_I \in \mathbb{R}_+$ on political advertising. Similarly, her challenger at the upcoming election is going to spend an amount $a_C \in \mathbb{R}_+$ on campaign activities. The probability that the incumbent wins the election is given by the continuous and differentiable function $\varphi : \mathcal{P} \times \mathbb{R}^2_+ \to [0, 1]$ that maps the policy choice and amounts spent on advertising into probabilities. The incumbent cares only about winning the election and simply maximizes the probability of doing so.

I assume that the function φ is increasing in policy on $[-1, 0)$ and decreasing in policy on $(0, 1]$ in any point where no candidate wins with certainty. The policy zero can thus be thought of as the policy most preferred by voters or at least the median voter. Implementing the policy zero is not enough for the incumbent to win the election with certainty, but at least guarantees a positive chance in the absence of campaign expenditures. That is, $0 < \varphi(0, 0, 0) < 1$. Furthermore, I require that φ is increasing in the campaign expenditure a_I of the incumbent and decreasing in the campaign expenditure a_C of the challenger, again in any point where no candidate wins with certainty.

Beyond these basic assumptions I need one restriction on the relative productivity of money spent by either candidate and on how the policy choice of the incumbent affects this productivity. Namely, I assume that campaign advertising is at least as effective for the incumbent as it is for the challenger: $\varphi(p, a, a) \geq \varphi(p, 0, 0)\ \forall p \in \mathcal{P}$. This assumption can be relaxed. In particular, one may want to allow for the possibility that the relative productivity of money spent by the incumbent decreases as policy becomes more extreme. This is possible as long as this decrease does not occur too quickly.

In general, the function φ can be thought of as representing some model of probabilistic voting where voters use the policies implemented by the incumbent to update their expectations about the utility they would receive in case the incumbent got re-elected. The influence of political advertising on voting behaviour is often interpreted as a purely psychological effect in the literature. It is also possible (if less easily so) to think of advertising as conveying actual information.

3.2 Interest Groups

Candidates do not have any funds of their own to spend on advertising. Instead they have to rely on interest groups for campaign contributions. Interest group $i \in \{L, R\}$ chooses to make donations a_I^i and a_C^i and has a utility function given by

$$U_i(p, a_I^i, a_C^i) = u_i(p) - a_A^i - a_B^i$$

where the function $u_i : \mathbb{R} \to \mathbb{R}$ is strictly decreasing (increasing) on $\mathcal{P}$ for $i = L$ $(i = R)$.

Interest groups make contributions according to contribution schedules communicated to candidates at the beginning of the game. I allow interest groups to condition their contributions on the policy choice of the incumbent and the contribution received by each candidate from the other interest group. Allowing for such general contracts can lead to problems of infinite regress. Suppose, for example, that conditional on a certain policy the schedules of the two interest groups take the following form: Interest group L commits to a contribution of x to the incumbent if the challenger receives no contributions and does not make any donations otherwise. Interest group R, on the other hand, contributes the same amount that the incumbent has received to the challenger. The final payments made under these contracts are then indeterminate. To prevent cases like this, I impose that the contributions made to any candidate by interest group i have to be a weakly increasing function of the payments received by candidates from other interest groups. This assumption is admittedly ad hoc, but required in order to ensure that the game is well-defined. I would like to stress that all of the equilibria presented in the paper are robust to deviations to contribution schedules that are not weakly increasing in donations by other interest groups, as long as it is possible to determine the contributions that result from this deviation. An additional restriction on contribution schedules is that no interest group can credibly promise donations greater than a group-specific amount A^i, which can be thought of as the total budget the group has available. To simplify notation I let A^i be the upper bound on promises made to each candidate separately instead of the upper bound on the sum of all promises.

I can now define the action spaces of interest groups formally. Let a_c^{-i} be the contribution received by candidate c from the interest group other than group i and let $\tilde{\mathcal{S}}^i$ be the set of all maps $s_i : \mathcal{P} \times \mathbb{R}_+^2 \to [0, A^i]^2$, with $s_{i,c}$ giving the contributions to candidate c specified by the map s_i. The action space of interest group i is then given by the set

$$\mathcal{S}^i \equiv \{s_i \in \tilde{\mathcal{S}}^i : \ s_{i,c}(p, a_I^{-i}, a_C^{-i}) \text{ weakly increasing in } a_I^{-i},\ a_C^{-i} \text{ for } c \in \{I, C\}\} \ .$$

The restrictions on schedules ensure that final transfers are always well defined: For a given policy choice p any vector of contracts defines an increasing self-map on the space $\times_{i \in \Theta}[0, D^i]^2$ equipped with the product order. Tarski's theorem therefore guarantees the existence of at least one fixed point. If more than one fixed point exists I pick the one for which all contributions are lowest (the infimum of the set of fixed points under the product order). This assumption is not restrictive: All equilibria derived in the paper rely on contribution schedules that have a unique fixed point for any given policy. For any vector of contribution schedules $\mathbf{s}$ let $a_{i,c}(p|\mathbf{s})$ be the contribution by interest group i to candidate c corresponding to this lowest fixed point. Also, let

$$a_i(p|\mathbf{s}) = a_{i,I}(p|\mathbf{s}) + a_{i,C}(p|\mathbf{s}) \ .$$

As advertising expenditures increase the probability of winning, candidates will always spend all of the donations they receive. I therefore denote by a_c the sum of contributions to candidate c as well as candidate c's expenditure.

3.3 Timing And Equilibrium

The timing of the game is simple: In the first stage all interest groups commit to a contribution schedule. Subsequently, the incumbent chooses a policy. Contributions are then made according to the previously announced schedules. Finally, the winner of the election is determined according to the function φ. I look for subgame perfect equilibria of this game, given by a vector of contribution schedules $\mathbf{s}^*$ and a function P^* that returns the policy choice of the incumbent for any possible pair of contribution schedules. I focus on pure strategy equilibria, as is standard in this literature.

The set of such equilibria is large as soon as more than one interest group is present. This stems from the fact that interest groups are almost unconstrained in their commitments to contributions at policies that occur out of equilibrium. Threatening to make sufficiently high donations can ensure that these donations never actually have to be carried out. It seems desirable to introduce at least some chance that interest groups will be held to their word. This is possible by requiring that equilibria are robust to an arbitrarily small chance that the incumbent turns out to be an irrational type, who announces a random policy without caring about her chance of being re-elected. This by itself, however, does not impose the desired degree of discipline on contribution schedules. Any particular out-of-equilibrium policy still occurs with zero probability and interest groups thus remain free to make "unreasonable" promises on a small set of platforms that have a huge impact on the incentives that the incumbent faces. This feature, in turn, rests entirely on the infinity of the policy space. I therefore introduce a perturbed version $G_\varepsilon^{\mathcal{P}_\Delta}$ of the contribution game G, which differs from the original game in two ways: First of all, the policy space is replaced by some finite subset $\mathcal{P}_\Delta$ of $\mathcal{P}$ and all functions (and function spaces) are appropriately restricted to $\mathcal{P}_\Delta$. Secondly, the incumbent is either a rational player with probability $1-\varepsilon$ or irrational with probability ε. An irrational incumbent does not care about her chances of winning the election and chooses a policy that—from the perspective of interest groups—is equally likely to be any point of the policy space $\mathcal{P}_\Delta$. I then consider only equilibria of the original game that are robust in the following sense:

Definition 1 (Robust contribution equilibrium)**.** *Consider an equilibrium $E = (\mathbf{s}^*, P^*)$ of the contribution game G. Let $\mathcal{P}_E$ be a finite subset of the policy space $\mathcal{P}$ that contains the points $P^*(\mathbf{s}^*)$ and zero. Let $\tilde{P}^*$ be such that $\tilde{P}^*(\mathbf{s}) \in \arg\max_{p\in\mathcal{P}_E} \varphi(p|\mathbf{s})$ with $\tilde{P}^*(\mathbf{s}^*) = P^*(\mathbf{s}^*)$. Denote by $\mathbf{s}^*|_{\mathcal{P}_E}$ the restriction of the equilibrium contribution schedules to the set $\mathcal{P}_E$.*
Then E is said to be a robust contribution equilibrium if, for any $\mathcal{P}_E$ and some corresponding $\tilde{P}^*$, there exists a positive probability $\bar{\varepsilon}$ such that for any $\varepsilon < \bar{\varepsilon}$ the strategy profile $(\mathbf{s}^*|_{\mathcal{P}_E}, \tilde{P}^*)$ is an equilibrium of the perturbed game $G_\varepsilon^{\mathcal{P}_E}$.

The inclusion of the equilibrium platforms in the discretised policy space greatly simplifies the definition and the application of the concept but is otherwise not essential.

Before proceeding to the description of the results, I need to introduce one more bit of notation. I define $\varphi(p|\mathbf{s})$ as the election probability of the incumbent under the policy choice p and given the contributions made at p under the vector of schedules $\mathbf{s}$.

4 One Interest Group

I will now describe the solution to the model in the case where interest group R is the only active lobby. This serves mainly as an introduction to the logic underlying the more general results in the following section. The equilibrium presented here can be derived from proposition 3 in the next section by setting $A^L = 0$. Therefore, no proofs will be given here.

In the absence of any contributions, the incumbent would maximise his probability of getting re-elected by choosing the policy zero. Interest group R would like to shift the chosen policy to the right. One way of achieving this would be to promise contributions to the campaign of the incumbent that can then be spent on political advertising. If the amount given is sufficiently high this could compensate for the votes lost due to the less voter-friendly policy. That is, the promised amount a would have to satisfy $\varphi(0,0,0) \leq \varphi(p,a,0)$ in order to make the incumbent implement the policy p. However, it would also be possible for interest group R to threaten the incumbent to give the same amount a to the challenger if the incumbent chooses any policy below some policy p'. The incumbent then chooses the policy p' as long as $\varphi(0,0,a) \leq \varphi(p',0,0)$, but the lobby does not have to make any actual contributions. Whether the policy p' is greater or smaller than the policy p depends on the shape of the function φ. For example, if campaign money is much more effective in the hands of the incumbent than when spent by the challenger, it will be true that $p > p'$.

It is not the case though that the interest group has to decide exclusively between making promises or threats. It may give money to the challenger at a certain policy, but threaten to withdraw this money and give it to the challenger at any policy closer to zero. But while the use of promises depends on their effectiveness, threats give influence at no cost and will therefore always be employed. Accordingly, the equilibrium policy and contribution is given by the solution to the maximisation problem

$$\max_{p,a} \quad u_R(p) - a$$
$$\text{s.t.} \quad \varphi(p, a, 0) \geq \varphi(0, 0, A^R) \ ,$$
$$0 \leq a \leq A^R \ .$$

The first constraint is a participation constraint that ensures that the incumbent is willing to locate at the targeted policy. The second constraint ensures that the lobby does not exceed its budget. The first order conditions for this problem can be rewritten to yield

$$u_R'(p) = -\frac{\partial\varphi/\partial p}{\partial\varphi/\partial a} \ .$$

The right-hand side of this condition is the increase in campaign contributions required to satisfy the participation constraint due to an increase in the targeted policy, as can be seen from the implicit function theorem. The condition therefore simply says that the marginal utility of policy must be equal to the marginal cost of achieving this policy at the optimum.

The interest group may rely entirely on threats, or may fully exploit its ability of making both threats and actual donations, or the equilibrium may lie anywhere in between these two extremes. If an observer was to attribute the difference between the equilibrium policy and the policy zero solely to contributions received by the incumbent, this would potentially give the impression of very high rates of return: There is no upper bound on the ratio between the utility gain of the interest group relative to the policy zero and equilibrium donations, as the latter may be arbitrarily small.

5 Two Interest Groups

I begin by deriving conditions that contribution schedules need to satisfy in an equilibrium that is robust in the sense of definition 1. In essence, these say that interest groups only commit to contributions where these are required to support the equilibrium policy choice of the incumbent. No interest group will promise contributions at policies that it prefers over the equilibrium policy, in particular. These would take the form of donations intended to make the incumbent deviate. If this fails there is no need to maintain these promises.

Lemma 1. *Consider an equilibrium* $(\mathbf{s}^*, P^*)$ *of the contribution game and let*

$p^* = P^*(\mathbf{s}^*)$. *This equilibrium is robust only if*

i) $a_{R,c}(p|\mathbf{s}^*) = 0$ *for* $c \in \{I, C\}$ *and any* $p > p^*$,

ii) $a_{L,c}(p|\mathbf{s}^*) = 0$ *for* $c \in \{I, C\}$ *and any* $p < p^*$,

iii) $\varphi(p, 0, 0) \leq \varphi(p^*|\mathbf{s}^*)$ *implies* $a_{i,c}(p|\mathbf{s}^*) = 0$ *for* $i \in \{L, R\}$ *and* $c \in \{I, C\}$.

iv) $\varphi(p, 0, 0) > \varphi(p^*|\mathbf{s}^*)$ *implies* $\varphi(p|\mathbf{s}^*) = \varphi(p^*|\mathbf{s}^*)$.

Proof. To show part *i*), consider any $p > P^*(\mathbf{s}^*)$ and assume $d_{R,c}(p|\mathbf{s}^*) > 0$. Suppose lobby R reduces all of its contributions at p to zero. If the payoff of the incumbent at p is now higher than her equilibrium payoff she would change her policy to p, making group R better off. If, on the other hand, the payoff of the incumbent at p remains at or below her equilibrium payoff then the equilibrium is not robust. To see this, note that there exists some finite subset $\mathcal{P}_p$ of the policy space that contains the policy p besides the policies $P^*(\mathbf{s}^*)$ and 0. For any $\varepsilon > 0$ the policy p is chosen by the irrational type of the incumbent with positive probability in the perturbed game $G_\varepsilon^{\mathcal{P}_p}$. As lobby R does not change the behaviour of the rational type of the incumbent by reducing its contributions at p to zero, but lowers its expected donations, it would prefer to do so. The necessity of the second part of the statement can be shown analogously.

Thus, for any policy other than p^* at most one interest group makes a contribution. Suppose $\varphi(p, 0, 0) < \varphi(p^*|\mathbf{s}^*)$ for some p and some interest group i makes a contribution at p. As in the previous paragraph, there exists some perturbed version of the game where the policy p is chosen with positive probability by the irrational type of the incumbent. Group i would therefore like to lower its contributions at p as long as this does not change the behaviour of the rational type of the incumbent. As i is the only lobby making a contribution at p and donations are increasing in the contributions of other groups, the donations of the second lobby have to remain at zero if group i reduces its contributions at p. The condition $\varphi(p, 0, 0) < \varphi(p^*|\mathbf{s}^*)$ is therefore sufficient to guarantees that i can lower its contributions at p to zero without inducing a deviation by the incumbent. This implies that part *iii*) is required for robustness.

Finally, the condition $\varphi(p, 0, 0) > \varphi(p^*|\mathbf{s}^*)$ entails that the incumbent must receive a donation at any such p, otherwise equilibrium would be violated. However, if it was the case that $\varphi(p|\mathbf{s}^*) < \varphi(p^*|\mathbf{s}^*)$, the continuity of the function φ enables the interest group making donations at p to lower these without affecting the behaviour of the incumbent. As in the previous paragraph, there exists a perturbed version of the game where the interest group also has the incentive to do so. This completes the proof. □

I now introduce the concept of a net contribution: A candidate is said to receive a net contribution if her payoff is higher than it would be if neither candidate received any contributions for a given policy choice of the incumbent. Formally, the incumbent is in receipt of a net contribution at p under a vector of schedules $\mathbf{s}$ if $\varphi(p|\mathbf{s}) > \varphi(p, 0, 0)$. Equivalently, the challenger receives a net contribution when $\varphi(p|\mathbf{s}) < \varphi(p, 0, 0)$.

Lemma 2. *In any robust contribution equilibrium and for any policy p, $a_L(p|\mathbf{s}^*) > 0$ only if the challenger receives a net contribution at the equilibrium policy.*

Proof. Let p^* be the policy choice and φ^* the payoff of the incumbent in equilibrium. Suppose the challenger does not receive a net contribution at p^*, that is $\varphi^* \geq \varphi(p^*, 0, 0)$. For any $p > p^*$ this implies $\varphi^* > \varphi(p, 0, 0)$. By lemma 1 it is therefore the case that neither group promises any contributions at any $p > p^*$ and consequently the payoff of the incumbent at any such p is strictly lower than the payoff $\varphi(p^*, 0, 0)$.

Now consider any policy $0 \leq p < p^*$. Due to lemma 1 it must be true that

$$\varphi(p|\mathbf{s}^*) = min\{\varphi^*, \varphi(p, 0, 0)\} \ .$$

It is therefore true that the payoff of the incumbent at any such p is strictly greater than $\varphi(p^*, 0, 0)$.

The last two paragraphs together show that the incumbent would deviate to a policy smaller than p^* if her payoff at p^* was lowered sufficiently. Consequently, interest group L must be making no contributions at p^*. Otherwise it could reduce these to zero, with one of two possible consequences: The equilibrium policy remains unchanged but group L saves on contributions, or the incumbent deviates to a preferable policy outcome. In the latter case, lemma 1 implies that lobby L makes no contributions at the new policy choice of the incumbent. This shows that dropping all contributions at p^* must be a profitable deviation for interest group L. □

The previous lemma says that if the election probability of the incumbent is not weighed down by donations to the challenger, then there is no risk that she will deviate to policies that are even further away from zero. These policies are bad for the prospects of the incumbent unless they bring contributions from interest group R. As was shown before, lobby R will not make such promises. Group L can therefore remain almost completely passive, in the sense made precise by the lemma.

The following proposition is the first main result of this section.

Proposition 1. *In any robust contribution equilibrium, only interest group R may give money and only to the incumbent.*

Proof. First, suppose that both interest groups make a contribution at the equilibrium policy p^*. By lemma 2 this requires that the challenger must be receiving a net contribution at p^*. This implies that the challenger must also be receiving a net contribution on some interval $(p^*, p]$ as the incumbent would deviate to one of these policies otherwise. By lemma 1 lobby L must be making these contributions. But then group L could reduce the donation to the challenger to zero at some policy $p^* + \varepsilon > p^*$, inducing the incumbent to deviate to this policy. For ε small enough this move must be profitable, as it implies a fixed reduction in contributions from $a_L(p^*|\mathbf{s}^*)$ to zero but a negligible loss in utility from policy. It is therefore impossible that both interest groups make a contribution in equilibrium.

Now suppose an interest group gives money to the challenger in equilibrium. It must then be the only group making positive donations. As contributions are increasing functions of contributions by other groups, lowering the donation to the challenger cannot change the amount of money donated by the second interest group. This move must consequently increase the payoff of the incumbent and therefore leaves her policy choice unchanged. This shows that no lobby gives money to the challenger.

To complete the proof, assume that group L gives money to the incumbent. As the challenger receives no contributions, this means that the incumbent would be receiving a net contribution. By lemma 2 this contradicts that lobby L would be making any donations. □

Donations to the challenger have the sole purpose of making a particular policy choice unattractive to the incumbent. There is therefore no reason to give money to the challenger at the equilibrium policy. Giving money to both candidates simultaneously is equally futile. Nevertheless, the complicated nature of contribution schedules makes it less than obvious that such things never occur. The preceding proposition shows that they don't.

There are two policies that provide bounds on the possible equilibrium choices of the incumbent. The first one is the policy furthest to the right of zero that interest group R can achieve purely by making threats while group L counters these threats by the highest possible promise of donations. I denote this policy by $\hat{p}$ and it is formally defined as the policy $p > 0$ that satisfies $\varphi(p, 0, 0) = \varphi(0, A^L, A^R)$. As will be shown below, $\hat{p}$ provides a lower bound on policy outcomes. An upper bound is given by $\check{p}$, defined as the policy $p > 0$ such that $\varphi(p, A^R, A^L) = \varphi(0, 0, A^R)$. For any policy to the right of $\check{p}$ interest group L can make the incumbent deviate to zero by giving A^L to the challenger, even if lobby R fully exploits its ability of using actual contributions and threats in order to influence the policy choice of the incumbent.

Proposition 2. *The equilibrium policy must be an element of the interval* $[\hat{p}, \bar{p}]$ *in any robust contribution equilibrium.*

Proof. Suppose there was an equilibrium where the incumbent chooses a policy $p^* < \hat{p}$. According to proposition 1 the challenger does not receive any contributions in equilibrium. Lemma 2 thus implies that $a_L(p|\mathbf{s}^*) = 0$ for any policy p and in particular that $a_L(p, 0, 0) = 0$ for any $p > p^*$. Now let interest group R deviate to the contribution schedule s' defined as follows: For some policy p' such that $p^* < p' < \hat{p}$ no candidate receives any donations from group R. For any policy other than p' group R gives A^R to the challenger. Under this schedule and the equilibrium schedule of lobby L the payoff of the incumbent from locating at p' is $\varphi(p', 0, 0)$. For the payoff from any other policy p it holds that

$$\begin{aligned} \varphi(p|s', s_L^*) &\leq \varphi(p, A^L, A^R) \\ &\leq \varphi(0, A^L, A^R) \\ &= \varphi(\hat{p}, 0, 0) \\ &< \varphi(p', 0, 0) \ , \end{aligned}$$

where the first line holds because the incumbent can at most receive A^L from interest group L at p, while the remaining lines use the assumption that the payoff of the incumbent is increasing towards zero and/or the definitions of p' and $\hat{p}$. This shows that interest group R can induce the incumbent to choose the policy p' without actually carrying out any donations, which must increase the utility of group R.

Now suppose there was an equilibrium where the incumbent chooses a policy $p^* > \check{p}$. As above, $a_L(p|\mathbf{s}^*) = 0$ for any policy p. If lobby L can lower the payoff of the incumbent at p^* sufficiently, the proof of lemma 2 together with the implication of proposition 1 that the challenger never receives a net contribution show that this must lead to a deviation of the incumbent to a policy smaller than p^*. To induce such a deviation group L can deviate to a schedule s' where L gives A^L to the challenger at p^* and otherwise commits to the same contributions as under the schedule s_L^*. In this case

$$\begin{aligned}
\varphi(p^*|s_R^*, s') &\leq \varphi(p^*, A^R, A^L) \\
&< \varphi(\check{p}, A^R, A^L) \\
&= \varphi(0, 0, A^R) \\
&\leq \varphi(0|\mathbf{s}^*) \ ,
\end{aligned}$$

where the first line holds because the incumbent can at most receive A^R from interest group R at p^*, the second line holds as the payoff of the incumbent is increasing towards zero, the third line uses the definition of $\check{p}$, while the final line uses the fact that $\varphi(0, 0, A^R)$ is the worst possible payoff for the incumbent at zero when group L makes no contributions. Lobby L can therefore achieve a policy smaller than p^* without carrying out any contributions, which must me profitable. □

So far it has been shown that the equilibrium policy must fall within a certain range the favours lobby R and if any contributions occur in equilibrium they will be given to the incumbent by interest group R. Interest group L, on the other hand, remains almost completely passive; it does not even offer contributions at out-of-equilibrium policies. Lobby R nevertheless has less influence than if group L was not present. This is because group L has the ability to react when group R increases its contributions in order to gain more influence. For example, consider the situation where L does not give any contributions to the incumbent at policies below the one chosen in equilibrium, while group R threatens to give some money to the challenger at said policies. R can then be unable to intensify these threats beyond the equilibrium level, because any additional money given would trigger contributions to the challenger from lobby L.

All of the above statements are conditional on equilibrium existence. I now conclude this section by constructing an equilibrium that always exists. To do so, define the policy $\bar{p}$ as the policy $p > 0$ such that $\varphi(p, A^R, A^L) = \varphi(0, A^L, A^R)$. Next, let the policy p^E be defined as the smallest policy p that is part of a solution to the maximisation problem

$$\begin{aligned}\max_{p,a} \quad & u_R(p) - a \\ \text{s.t.} \quad & \varphi(p, a, 0) = \varphi(0, A^L, A^R) \ , \\ & \hat{p} \leq p \leq \bar{p} \ , \\ & 0 \leq a \leq A^R \ .\end{aligned}$$

Accordingly, define the contribution schedule s_R^E such that at p^E the incumbent receives the donation a that solves $\varphi(p^E, a, 0) = \varphi(0, A^L, A^R)$, while at any other policy p the challenger receives the smallest possible contribution a that ensures that $\varphi(p, 0, a) \leq \varphi(0, A^L, A^R)$. Furthermore, group R commits to increasing its donation to the incumbent at p^E to A^R if lobby L should make any contributions. Similarly, it commits to contributing A^R to the campaign of the challenger at any $p \neq p^E$ if lobby L should make any contributions.

The contribution schedule s_L^E can then be defined by

$$s_{L,C}^E(p, a_I^R, a_C^R) = \begin{cases} A^L & \text{if } p^E \neq p > \hat{p} \text{ and } a_I^R > 0 \\ 0 & \text{otherwise} \end{cases}$$

and

$$s_{L,I}^E(p, a_I^R, a_C^R) = \begin{cases} A^L & \text{if } p = 0 \text{ and } a_C^R > s_{R,C}^E(0,0,0) \\ 0 & \text{otherwise .} \end{cases}$$

Proposition 3. *The strategy profile* $(\mathbf{s}^E = (s_L^E, s_R^E), P^E)$ *is an equilibrium for some* P^E *such that* $P^E(\mathbf{s}^E) = p^E$*.*

Proof. The policy p^E is an optimal choice for the incumbent under the schedules s_L^E and s_R^E by construction. It remains to be shown that no interest group wants to deviate.

To see that interest group R cannot achieve a better outcome, first note that it is impossible for R to lower the payoff of the incumbent at 0 below $\varphi(0|\mathbf{s}^E)$. As $\varphi(0, 0, 0) > \varphi(0, A^L, A^R)$, the definition of the schedule s_R^E implies that the challenger receives a contribution from R at zero such that $\varphi(0|\mathbf{s}^E) = \varphi(0, A^L, A^R)$. Lowering the payoff of the incumbent at zero would require increasing the donation to the challenger, but this would immediately cause lobby L to give A^L to the incumbent. The lowest payoff R can thus achieve is $\varphi(0, A^L, A^R)$, which is the equilibrium level.

There are then three possible cases to consider: Interest group R could try to move the policy of the incumbent to a point below or at $\hat{p}$, to a policy in the set $(\hat{p}, \bar{p}]$, or to an even greater policy. As the definition of p^E implies that group R is either already making the incumbent locate at $\hat{p}$ for free or prefers to pay to make her choose a greater policy, any policy smaller than or at $\hat{p}$ cannot be better for R even if it can be achieved for free. Next, consider any policy in the set $(\hat{p}, \bar{p}]/p^E$. For any such policy p it is true that $s_R^E(p, 0, 0) = 0$ by the definition of s_R^E as

$$\varphi(p, 0, 0) < \varphi(\hat{p}, 0, 0) = \varphi(0, A^L, A^R) \ . \tag{1}$$

By the definition of s_L^E this means that any contribution at p to the incumbent by group R will cause lobby L to give a donation of A^L to the challenger. However, in order to make the incumbent choose the policy p she would have to receive a contribution, as can be seen from condition (1). Suppose then that interest group R could make a contribution a' such that $\varphi(p, a', A^L) > \varphi(0, A^L, A^R)$, as would be required to make the incumbent locate at p. This donation must clearly be greater than the donation a'' necessary to achieve $\varphi(p, a'', 0) = \varphi(0, A^L, A^R)$. It therefore holds that

$$\begin{aligned} u_R(p) - a' &< u_R(p) - a'' \\ &\leq u_R(p^E) - s_{R,I}^E(p^E, 0, 0) \ , \end{aligned}$$

where the second line holds due to the definition of p^E. This shows that such a deviation cannot be profitable.

For the third case, it is clear that inducing the incumbent to locate at some $p > \bar{p}$ is impossible. As in the previous paragraph, this would require a donation to the incumbent, which would provoke a reaction from group L. Accordingly, the highest payoff that would be possible for the incumbent is $\varphi(p, A^R, A^L)$ for which it holds that $\varphi(p, A^R, A^L) < \varphi(\bar{p}, A^R, A^L) = \varphi(0, A^L, A^R)$.

Finally, it needs to be shown the interest group L cannot improve on the equilibrium outcome. To do so, it would have to change the policy choice of the incumbent, which in turn would require L to raise the payoff of the incumbent at some $p \neq p^E$ above $\varphi(0, A^L, A^R)$ or to lower the payoff at p^E. The former could only be achieved through giving money to the incumbent, which would be countered by group R with a donation of A^R to the challenger. If follows that the highest possible payoff lobby L could achieve through a donation to the incumbent at p is $\varphi(p, A^L, A^R) < \varphi(0, A^L, A^R)$. Lowering the payoff of the incumbent at p^E is equally impossible as group R reacts to any donations to the challenger and thus $\varphi(p^E, A^R, A^L) \geq \varphi(\bar{p}, A^R, A^L) = \varphi(0, A^L, A^R)$. □

As the final step, I verify that the equilibrium established above is also a robust contribution equilibrium.

Proposition 4. *If the strategy profile* $(\mathbf{s}^E, P^E)$ *is an equilibrium, then it is a robust contribution equilibrium.*

Proof. Consider some finite subset $\mathcal{P}_E$ of the policy space that contains the policies zero and p^E and a strategy profile $(\mathbf{s}^E|_{\mathcal{P}_E}, \tilde{P}^E)$, as required in the definition of a robust contribution equilibrium. The arguments of the proof of proposition 3 can also be applied to the perturbed game to show that the incumbent must locate at p^E under any schedule that interest group L can propose. L consequently has no profitable deviations. Group R, on the other hand, cannot reduce any contributions without causing the incumbent to deviate by the definition of the schedule s_R^E and because the policy zero is included in $\mathcal{P}_E$. It therefore remains to check that R would not want to deviate to some schedule that induces a different policy choice.

The proof of proposition 3 implies that

$$u_R(p') - a' < u_R(p^E) - s_{R,I}^E(p^E, 0, 0) \tag{2}$$

for any policy $p' > \hat{p}$ that interest group R may be able to achieve and the contribution a' that would be required to do so. Now, p^E has been defined such that either $p^E = \hat{p}$, in which case $s_{R,I}^E(p^E, 0, 0) = 0$ as this policy can be achieved for free, or it must be the case that

$$u_R(\hat{p}) < u_R(p^E) - s_{R,I}^E(p^E, 0, 0) \ .$$

As $\hat{p}$ is also the largest policy that can be achieved for free, this shows that condition (2) actually applies to any policy that interest group R can induce against the schedule s_L^E.

For any schedule s of interest group R that induces a deviation of the rational incumbent to some policy p', the utility of interest group R in the perturbed game $G_\varepsilon^{P_E}$ can be written as

$$\varepsilon \ \frac{1}{|P_E|} \sum_{p \in P_E} \left[u_R(p) - a_R(p|s, s_L^E) \right] + (1 - \varepsilon) \left[u_R(p') - a' \right] \ ,$$

using the same notation as in the previous paragraph. This utility can be no greater than

$$\varepsilon \, \frac{1}{|P_E|} \sum_{p \in P_E} u_R(p) + (1-\varepsilon)\,[u_R(p') - a'] \ .$$

The difference between the equilibrium utility and this last expression is

$$\varepsilon \, \frac{1}{|P_E|} \sum_{p \in P_E} \left[-s_R^E(p,0,0)\right] + (1-\varepsilon)\left[(u_R(p^E) - s_{R,I}^E(p^E,0,0)) - (u_R(p') - a')\right] \ .$$

This difference converges to

$$(u_R(p^E) - s_{R,I}^E(p^E,0,0)) - (u_R(p') - a')$$

as ε approaches zero, which is positive by condition (2). This shows that there is no profitable deviation for the incumbent from the schedule $s_R^E|_{\mathcal{P}_E}$ for ε sufficiently small, establishing the robustness of the equilibrium $(\mathbf{s}^E, P^E)$. □

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